Instructions: Try these problems from the textbook as practice (answers below).

Problems

3-8. Sketch the plane curve with the given vector equation, find $\mathbf{r}'(t)$, and sketch the position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ for the given value of t.

3. $\mathbf{r}(t) = \langle t-2, t^2+1\rangle, t=-1$
5. $\mathbf{r}(t) = \langle e^{2t}, e^t\rangle, t=0$
7. $\mathbf{r}(t) = 4\sin(t)\mathbf{i} - 2\cos(t)\mathbf{j}, t=3\pi/4$

9-16. Compute the derivative of the vector function.

9. $\mathbf{r}(t) = \langle \sqrt{t-2}, 3, \frac{1}{t^2}\rangle$
11. $\mathbf{r}(t) = t^2\mathbf{i} + \cos(t^2)\mathbf{j} + \sin^2(t)\mathbf{k}$
13. $\mathbf{r}(t) = t\sin(t)\mathbf{i} + e^t\cos(t)\mathbf{j} + \sin(t)\cos(t)\mathbf{k}$
15. $\mathbf{r}(t) = \mathbf{a} + t\mathbf{b} + t^2\mathbf{c}$

17-20. Compute the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of t.

17. $\mathbf{r}(t) = \left\langle t^2-2t, 1+3t, \frac{1}{3}t^3 + \frac{1}{2}t^2\right\rangle, t=2$
19. $\mathbf{r}(t) = \cos(t)\mathbf{i} + 3t\mathbf{j} + 2\sin(2t)\mathbf{k}, t=0$

21. Compute the unit tangent vector $\mathbf{T}(t)$ at the point $(2,-2,4)$ on the curve $\mathbf{r}(t) = \langle t^3+1, 3t-5, 4/t\rangle$.

23. If $\mathbf{r}(t) = \langle t^4, t, t^2\rangle$, find $\mathbf{r}'(t)$, $\mathbf{T}(1)$, $\mathbf{r}''(t)$, and $\mathbf{r}'(t)\times\mathbf{r}''(t)$.

25-28. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

25. $x = t^2+1$, $y = 4\sqrt{t}$, $z = e^{t^2-t}$; $(2,4,1)$
27. $x = e^{-t}\cos(t)$, $y = e^{-t}\sin(t)$, $z = e^{-t}$; $(1,0,1)$

29. Find a vector equation for the tangent line to the curve of intersection of the cylinders $x^2+y^2=25$ and $y^2+z^2=20$ at the point $(3,4,2)$.

37-42. Evaluate the integral.

37. $\displaystyle\int_0^2 (t\mathbf{i} - t^3\mathbf{j} + 3t^5\mathbf{k})\,dt$
39. $\displaystyle\int_0^1 \left(\frac{1}{t+1}\mathbf{i} + \frac{1}{t^2+1}\mathbf{j} + \frac{t}{t^2+1}\mathbf{k}\right)\,dt$
41. $\displaystyle\int \left(\frac{1}{t^2+1}\mathbf{i} + te^{t^2}\mathbf{j} + \sqrt{t}\mathbf{k}\right)\,dt$

43. Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = 2t\mathbf{i} + 3t^2\mathbf{j} + \sqrt{t}\mathbf{k}$ and $\mathbf{r}(1) = \mathbf{i}+\mathbf{j}$.

49. If $\mathbf{u}(t) = \langle\sin(t), \cos(t), t\rangle$ and $\mathbf{v}(t) = \langle t, \cos(t), \sin(t)\rangle$, find $\dfrac{d}{dt}[\mathbf{u}(t)\cdot\mathbf{v}(t)]$.

51. Find $f'(2)$, where $f(t) = \mathbf{u}(t)\cdot\mathbf{v}(t)$, $\mathbf{u}(2) = \langle 1,2,-1\rangle$, $\mathbf{u}'(2) = \langle 3,0,4\rangle$, and $\mathbf{v}(t) = \langle t, t^2, t^3\rangle$.

Answers

3. $\mathbf{r}'(t) = vect1, 2t$
5. $\mathbf{r}'(t) = \left\langle 2e^{2t}, e^t \right\rangle$
7. $\mathbf{r}'(t) = 4\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j}$
9. $\mathbf{r}'(t) = \left\langle \dfrac{1}{2\sqrt{t-2}}, 0, -\dfrac{2}{t^3} \right\rangle$
11. $\mathbf{r}'(t) = \left\langle 2t, -2t\sin(t^2), 2\sin(t)\cos(t) \right\rangle$
13. $\mathbf{r}'(t) = \left\langle t\cos(t) + \sin(t), e^t(\cos(t) - \sin(t)), \cos^2(t) - \sin^2(t) \right\rangle$
15. $\mathbf{r}'(t) = \mathbf{b} + 2t\mathbf{c}$
17. $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}(t)\|} = \frac{1}{7}\langle 2, 3, 6\rangle$
19. $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}(t)\|} = \frac{1}{5}\langle 3, 4\rangle$
21. $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}(t)\|} = 34^{-1/2}\langle 3, 3, -4\rangle$
23. $\mathbf{r}'(t) = \left\langle 4t^3, 1, 2t \right\rangle$, $\mathbf{T}(1) = \dfrac{\mathbf{r}'(1)}{\|\mathbf{r}(1)\|} = 21^{-1/2}\langle 4, 1, 2\rangle$, $\mathbf{r}''(t) = \left\langle 12t^2, 0, 2 \right\rangle$, $\mathbf{r}'(t) \times \mathbf{r}''(t) = \left\langle 2, 16t^3, -12t^2 \right\rangle$.
25. $x = 2 + 2t, y = 4 + 2t, z = 1 + t$
27. $x = 1 - t, y = t, z = 1 - t$
29. $\mathbf{r}(t) = (3 - 4t)\mathbf{i} + (4 + 3t)\mathbf{j} + (2 - 6t)$
37. $\langle 2, -4, 32\rangle$
39. $\left\langle \ln(2), \frac{\pi}{4}, \frac{1}{2}\ln(2) \right\rangle$
41. $\left\langle \arctan(t), \frac{1}{2}e^{t^2}, \frac{2}{3}t^{3/2} \right\rangle + \mathbf{C}$
43. $t^2\mathbf{i} + t^3\mathbf{j} + \frac{2}{3}(t^{3/2} - 1)\mathbf{k}$
49. $2t\cos(t) + 2\sin(t) - 2\cos(t)\sin(t)$
51. 35